

Field Noise Effects in NMR

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Introduction

In all branches of NMR, the quality of the main magnetic field is of utmost importance. The principle characteristics of the field are its magnetic induction value, determining the Larmor frequencies of the observed nuclei, its homogeneity across the sample, determining the resolution with which one can resolve different components of the spin-system Hamiltonian, and its stability. The latter parameter, though widely recognized as being of utmost importance, is often given for granted and its effects on NMR signals are usually glossed-over in a qualitative way. Somehow, it is felt that to make the field stable enough is the task of the instrument manufacturer, ignoring the fact that absolute stability is impossible to reach. The Author has undertaken a systematic study of field-noise effects on various types of NMR signals. The study shows that in order to predict such effects, one must first estimate the field-noise auto-correlation function (mere noise rms amplitude is of little help). However,

once the autocorrelation function is known, it is all one needs to compute the effects of field instability on accumulated NMR data.

This is by itself a surprise since the evaluation of single-scan NMR signal statistics is considerably more complex and requires a more detailed knowledge of the field-noise properties. The treatment based on the field-noise autocorrelation function, to the extent to which it can be applied, is very convenient because it enables one to handle both random noise and periodic - or quasi-periodic fluctuations - within the same theoretical framework. Some of the results of the quantitative analysis are novel and surprising. In particular, this applies to the striking resonant effects of periodic and quasi-periodic noise on the CPMG decays. The newly-gained insights, even those regarding much less dramatic effects, are important since they regard such ubiquitous problems as FID shape distortions, distortions of spectral lines, refocussing efficiency of many multi-pulse sequences, t_1 noise in 2D spectroscopy, systematic errors in NMR relaxometry, artifacts in MRI images, etc.

Magnetic field noise arises from many sources

- **Magnet system components** (direct effect of current generator, active field stabilizer, shims, NMR lock system, field gradients, ...)
- **Instrument console** (induction from power wiring, transformers, relays, ...)
- **Environment** (induction from electric wiring, transformers, electric motors, mains stabilizers, air-conditioning, etc.)
- **Sample motion** across magnetic field inhomogeneities (sample rotation, vibrations)

Some of the field-noise components are **aperiodic** (random), while others are **periodic** with frequencies which are multiples of the AC power-mains frequency.

Quasi-periodic field fluctuations, such as field fluctuations induced by imperfect sample rotation, also need to be taken into account.

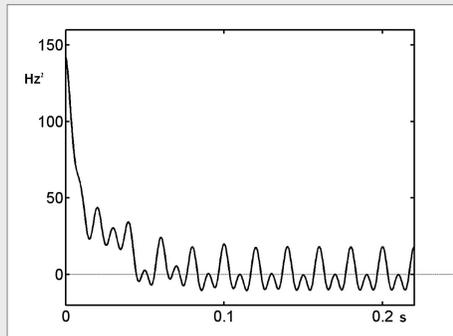
!!! Noiseless magnetic field does not exist !!!

Magnetic field-noise characteristics

- Mean value B_0
- Standard deviation σ
- Auto-correlation function $C(\tau)$, $C(0)=\sigma^2$
- Normalized correlation function $c(\tau)=C(\tau)/C(0)$

The Figure illustrates the autocorrelation function of a field-noise containing the following components (rms values):

- 10 Hz of white noise,
- 3 Hz of mains-induced brum (50 Hz frequency),
- 3 Hz of ripple from the instrument (100 Hz frequency),
- 5 Hz sample-rotation modulation (spinning at 30 Hz $\pm 10\%$).



Typical field noise/ripple rms magnitudes

- Straight wire carrying 1A AC current positioned 2m from an unshielded sample: 2.1289 Hz, induced ripple (exact value)
- Wire loop around a 5m x 5m room, supplying 300W for the room lights: about 7 Hz, induced ripple
- Room temperature shims (HR): 1-10 Hz, random
- Sample rotation (HR): 1-10 Hz, quasi-periodic
- MRI field gradients: 10-1000 Hz, random
- FFC-NMR magnet system: 100-1000 Hz, random
- 10 kW mains stabilizer distant 5m from an unshielded sample: up to 1000 Hz, induced ripple

Basic types of noise autocorrelation functions

- A. Random white** $C(\tau) = \sigma^2 \exp(-\tau/T_m)$, where T_m is the field-noise correlation time.
- B. Periodic** $C(\tau) = \sigma^2 \cos(-2\pi\tau/T_p)$, where T_p is the period of the field fluctuations.
- C. Quasi-Periodic** $C(\tau) = \sigma^2 \cos(-2\pi\tau/T_p) \exp(-\Delta\tau)$, where 2Δ is the half-height width of the Lorentzian spectral band of the field fluctuations. We will write $\Delta = k(2\pi/T_p)$, where k describes the uncertainty of the fluctuation frequency as a fraction of its mean value.

Effect of field-noise on averaged FID's

The final effect on averaged FID's can be described as a multiplication by a weight function $G(t)$ which is determined solely by $C(\tau)$. Though the dependence may be complicated, its main features are easy to discuss and grasp.

Copresence of uncorrelated noise components

When several, mutually uncorrelated field-noise components are present, the weighing function $G(t)$ becomes a product of factors, each corresponding to one of the individual noise components.

Principal field-noise effects on averaged NMR signals

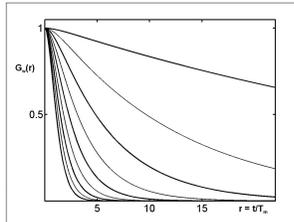
- **In High-Resolution NMR spectroscopy**
 - Characteristic broadening of spectral peaks, often accentuating their feets.
 - Limits to shimming the field homogeneity and interference with automatic shimming algorithms.
 - Sidebands at integer multiples of the mains frequency.
 - Sample rotation effects: sidebands, plus extra signal broadening.
 - Performance impairment of multi-pulse sequences (particularly those with multiple refocussing).
 - Dominant source of t_1 -noise in 2D and 3D spectroscopies.
- **In NMR relaxometry** (both classic and Fast-Field-Cycling, where field noise is particularly large)
 - Characteristic deformation of FID shapes.
 - Severe distortions of CPMG decays at particular values of the spacing between consecutive echos.
- **In PFGSE techniques** (self-diffusion measurements)
 - Contribution to experimental errors strongly dependent upon the echo-delay setting.
- **In magnetic resonance imaging (MRI)**
 - Image resolution impairment.
 - Image fringes related to the mains frequency.

A common symptom: **reduced efficiency of data averaging (accumulation).**

All these effects are now quantitatively understood. Explicit formulae link all of them to the field-noise auto-correlation function.

Effects of random noise (case A) on averaged FID's

The FID's are multiplied by a weighing functions $G(t) = G_w(r)$ which depends upon the normalized time ratio $r = t/T_m$ and upon the product $w = \gamma\sigma T_m$ (with γ being the gyromagnetic ratio). The plotted curves correspond to $w = 0.1$ (top), 0.2, 0.3, ..., 0.9 (bottom). When $t < T_m$, we have $G(t) = \exp[-(\gamma\sigma)^2 t^2/2]$, corresponding to spectral convolution with a Gaussian of standard width $2(\gamma\sigma)$ [rad]. In the opposite extreme, when $t > 10T_m$, the Gaussian part of $G(t)$ is close to 1 and the FID is in essence multiplied by the exponential $G(t) = \exp[-(\gamma\sigma)^2 T_m t]$, corresponding to spectral convolution with a Lorentzian line of half-height width $2(\gamma\sigma)^2 T_m$ [rad]. In the intermediate region we have a spectral line-shape deformation tending towards a Lorentzian central peak with a broader Gaussian 'hump'. The transition between the two regions, however, is too smooth for a neat visual distinction of the two components. In practice, whether the convolution is Gaussian, Lorentzian or intermediate depends upon the duration of the FID with respect to T_m .

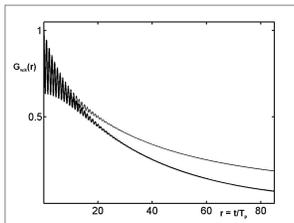


Effects of quasi-periodic fluctuations (case C) on averaged FID's

The weighing functions $G(t) \equiv G_{w,k}(r)$ now depend on $r = t/T_p$, $w = \gamma\sigma T_p$, and k . The formulae for $G_{w,k}(r)$ involve two components: an exponential factor and a damped oscillation (though not a harmonic one). The bold curve has been calculated for $w = 3$ and $k = 0.02$ (just $\pm 2\%$ uncertainty in the frequency of the fluctuations).

The exponential term implies a Lorentzian broadening of $\Delta = kw^2/T_p$ which affects all spectral lines. The cyclic factor gives rise to modulation sidebands in the spectra at multiples of the field modulation frequency. The relative intensities of the sidebands depend on both w and k and the damping of the oscillations implies that the sidebands are subject to an additional broadening of $\delta = 2(kw)$.

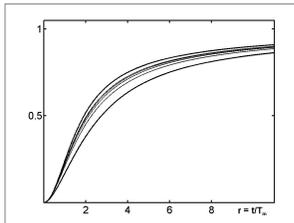
The thin curve in the Figure regards the case of field modulation due to sample rotation with averaging over various voxels of a cylindrical sample (second averaging). Decaying asymptotically as const/r , it gives rise to hump-broadening of spectral lines.



Effects of random noise (case A) on averaged CPMG decays

The final effect can be expressed as an anomalous contribution to the observed decay rate R_2 (ideally, $R_2 = 1/T_2$). Quantitatively, the anomalous contribution equals $wf_n(r)$, where n is the echo number, $w = (\gamma\sigma)^2 T_m$, and $r = \tau/T_m$, τ being half the interval between consecutive echos. In the Figure on the right, the dimensionless functions $f_n(r)$ are plotted against r for $n = 1$ (lowest), 2, 3, 4, 5, and 1000.

It turns out that the dependence of $f_n(r)$ upon n is quite modest and, in any case, converges to a common value after very few echos (this justifies the interpretation as a R_2 contribution). Given a fixed T_m , it is very important to keep the value of τ as small as possible. Like many other contributions to the CPMG decay rate, the anomalous "noise contribution" in fact vanishes when $\tau \rightarrow 0$.



Effects of periodic fluctuations (case B) on averaged CPMG decays

The graph shows a theoretical simulation of the effects of periodic field modulation upon averaged CPMG decays when τ (half the interval between consecutive echos) is close to $T_p/4$ (T_p being the field-modulation period). We assume $R_2 = 0.5 \text{ s}^{-1}$, $T_p = 20 \text{ ms}$ (European mains power frequency), and $w = (\gamma\sigma)^2 T_p = 20 \text{ rad}^2/\text{s}$ which, for protons, implies a field-modulation amplitude of about 0.12 μT (a smaller-than-usual value for most laboratory environments).

The plotted curves differ by the ratio $r = \tau/T_p$ whose values are, from bottom up, 0.25, 0.2475, 0.245 (thick), 0.24, 0.23, 0.2 and 0.1 (indistinguishable from an unperturbed decay). The marked resonant distortions for τ settings close to 5 ms have been often observed and even tackled theoretically (Allerhand A., Effect of Magnetic Field Fluctuations in Spin-Echo NMR Experiments, Rev.Sci.Instruments 41,269,1970). The present treatment is much more complete and it is the first one to be fully born out by experiment.

